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Circular Motion & Gravitation

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Circular Motion

Quantities Related To Circular Motion:

Consider a body moving around a circle of radius r with a constant speed v. In time Δt, the radius sweeps an angle $\Delta\theta$.

Time period (T): The time required by the body to complete a revolution.

Angular displacement (θ): The angle swept by the radius. Unit is radian.

Angular velocity (ω): The angle swept per unit time. Unit is radian s^{-1}.
$\omega = \Delta\theta/\Delta t$
In terms of time period,
$\omega = 2\pi/T$

Frequency (f): Number of revolutions per unit time.
$f = 1/T$
Angular velocity and frequency are related as $\omega = 2\pi f$

Angular Velocity And Speed:

The speed (v) and angular velocity (ω) of an object moving along a circle of radius r are related as $v = r\omega$

E.g.: A large clock on a building has a minute hand that is 4.2m long.
Calculate:
a) the angular speed of minute hand.
b) the angular displacement in the time periods
i) 12:00 to 12:20
ii) 12:00 to 14:30
c) the linear speed of tip of minute hand.

Answer.

a) The minute hand takes 60 mins = 3600 s for complete revolution.

Angular velocity = $2\pi/3600$ = rad s^{-1}

b) i) 20 minutes is a third of a complete revolution. So, the angular displacement is (2/3)π radi.

ii) 12:00 to 14:30 is 2 complete revolutions and a half revolution. So, the angular displacement is 2(2π) + π = 5π radi.

c) the linear speed is $v = r\omega$ = 4.2 × 0.00175 = $0.0073 ms^{-1}$

Centripetal Acceleration:

If the speed of an object remains constant during the travel in a circle, then it is called uniform circular motion. Although the speed remains constant, the velocity changes due to change in direction. So, there must be acceleration. This acceleration is called centripetal acceleration and is directed **towards the centre**. The magnitude of this acceleration is $a = v^2/r$.

Centripetal Force:

The force that causes centripetal acceleration is centripetal force. The magnitude of this force is mass times the magnitude of centripetal acceleration.

$F_c = mv^2/r$

The work done by the centripetal force is zero. Reason this using basic formula of work.

Examples Of Circular Motion:

1. Rotating a mass on a string on the earth (vertical):
   The critical velocity should be such that when the mass is at the highest position, the tension in the string is greater than (at least) zero.
2. Looping
   Consider a car looping a vertical circle. For the car to complete the circle, the normal reaction should be greater than (at least) zero at the top most point.

Find the least speed with which the masses should set out at the bottom in above two examples to complete the circle using law of conservation of energy.

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3. Car on a circular track:
   A car is moving on a circular track. The centripetal force is provided by frictional force whose maximum value is μR. When the speed becomes higher than a critical value such that the required centripetal force exceeds maximum fiction, the car slips.

E.g.: A hammer thrower in an athletics competition swings the hammer on its chain round 7.5 times in 5.2s before releasing it. The hammer describes a circle of radius 4.2m and has a mass of 4.0kg. Assume that the hammer is swung in a horizontal circle and that the chain is horizontal.

a) Calculate, for the rotation:

(i) the average angular speed of the hammer

(ii) the average tension in the chain.

b) Comment on the assumptions made in this question.

Answer.

a) i) The average angular speed =

$total\ angular\ displacement/total\ time = (7.5\times2\pi)/5.2 = 9.1\ rad\ s^{-1}$

ii) Average tension = $mr\omega^2 = 4 \times 4.2 \times 9.1^2 = 1400N$

b) The thrower usually inclines the plane of the circle at about 45° to the horizontal in order to achieve maximum range. Even if the plane were horizontal, then the weight of the hammer would contribute to the system so that a component of the tension in the chain must allow for this. Both assumptions are unlikely.

Gravitation

Newton's Universal Law Of Gravitation:

Every single point mass attracts every other point mass with a force that is directly proportional to the product of their masses and inversely proportional to the square of distance between them.

$F \propto m_1 m_2 / r^2$

$F = G m_1 m_2 / r^2$

G is called the **universal gravitational constant** and has a value $6.6742 \times 10^{-11} m^3 kg^{-1} s^{-2}$.

Gravitational Field And Field Strength:

The **gravitational field** of an object is the region in which another object experiences gravitational force. The gravitational field due to any mass extends to infinite space but its value becomes insignificant in large distances due to inverse square proportionality to distance.

Gravitational field strength of a mass is the force acting on a unit mass due to the mass. It is a vector and the combined field due to two masses can be calculated by adding the fields due to individual objects vectorially.

Due to a point mass of mass M, the gravitational field is GM/r^2.

Spherical objects can be considered point masses located at the center of the sphere in calculation of field.

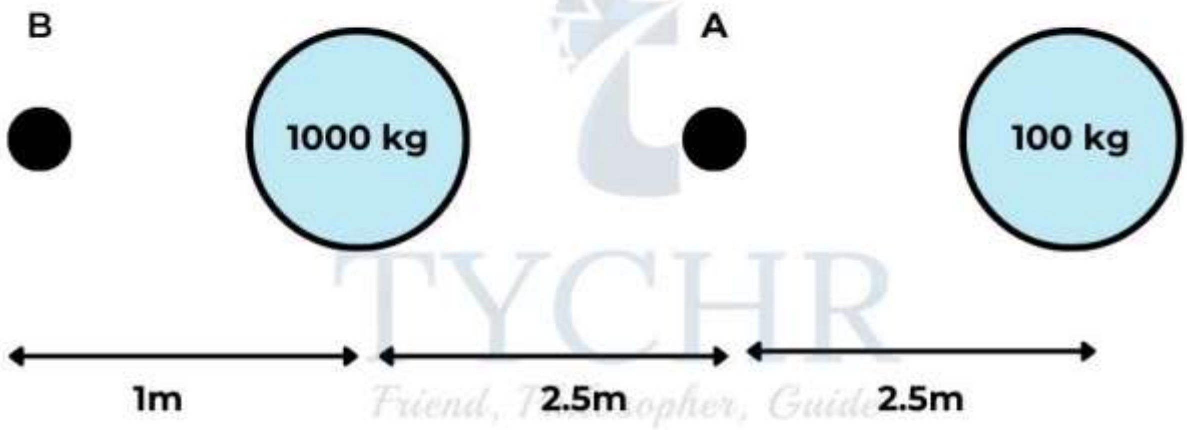
Close to earth, the field can be considered uniform with the strength of the field being equal to that on the surface.

Field Lines:

Field lines are lines drawn in the direction that a mass would accelerate if placed in the field.

They help to visualise the field.

E.g.: Calculate the field strength due to two masses at points A and B.



Answer. The field at A = magnitude of strength due to 1000kg – magnitude of strength due to 100kg = $(G1000/2.5^2)$ - $(G100/2.5^2)$= 9.63 × $10^{-9} Nkg^{-1}$
Field at B = magnitude of strength due to 1000kg + magnitude of strength due to 100kg = $(G1000/\ 1^2)$ - $(G100/6^2)$= 6.69 × $10^{-8} Nkg^{-1}$

Gravitational Potential:

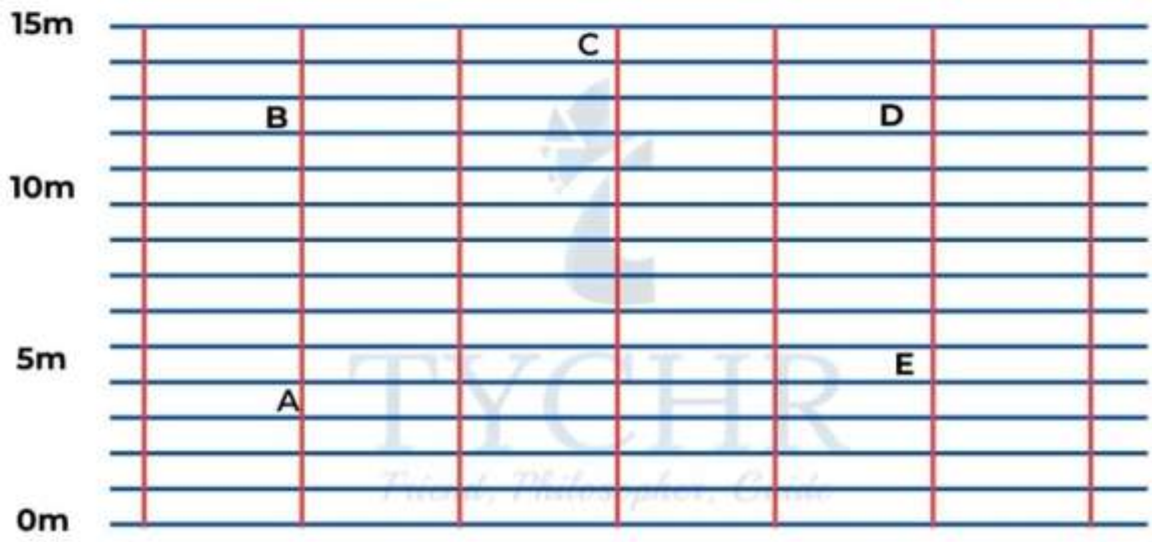
The gravitational potential at a point is the **work done per unit mass** in taking a point mass from zero potential (reference) to the

point of concern. The potential energy of a body is the potential at that point multiplied by the mass of the body.
In a uniform field (F) like that of one near the surface of the earth, the gravitational potential is equal to Fd, where d is the component of distance along the field.

Equipotential Lines:

The lines along which potential energy does not change are called equipotential lines. When a body is moved along one of these lines, work need not be done because they are of same potential.

Drawing equipotential lines for masses in three dimensions gives equipotential surfaces.



Equipotential and field lines near the surface of the earth.

The potential is a scalar quantity. So, the potential at a point due to two masses is the sum of potentials due to the two masses.

The equipotential lines are perpendicular to field lines. Find a reason for this based on the definition of work.

Potential Energy Due To Point Mass:

Consider a mass m present in a field due to a mass M. The potential energy of the mass m is the work done in bringing the point mass from infinity to a point at a distance r from M. this can be obtained by integrating force with distance from infinity to the required point.

This gives $W = -GMm/r$.

The potential due mass M is $W/m = -GM/r$.

Gravitational Field Inside A Planet:

The gravitational field inside a uniform planet of density , radius R at a distance r from centre is given by
$g' = 4\pi G\rho r/3$

Escape Speed:

The escape speed of a planet is the minimum speed with which a body projected would escape the gravitational field of the planet.
$v_e = \sqrt{(2GMm/R)}$

Energy Of A Satellite:

The total energy of a satellite of mass m encircling a planet of mass M in a circular path of radius R is $-GMm/2R$

Kepler's Third Law:

The orbital period T of a planet revolving around a sun in a circular path of radius r is related to orbital radius as

$T^2 = 4\pi^2 r^3 / GM$

$T^2 \propto r^3$

E.g.: The gravitational time period for earth around the sun is seconds. Calculate the time period for mars.
(radius of Earth orbit = 1.5×10^{11} m;
radius of Mars orbit = 2.3×10^{11} m)

Answer. $T^2 \propto r^3$

Which gives time period for mars = $T_E\sqrt{\frac{r_m^3}{r_e^3}} = 6.1 \times 10^7$ s.

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