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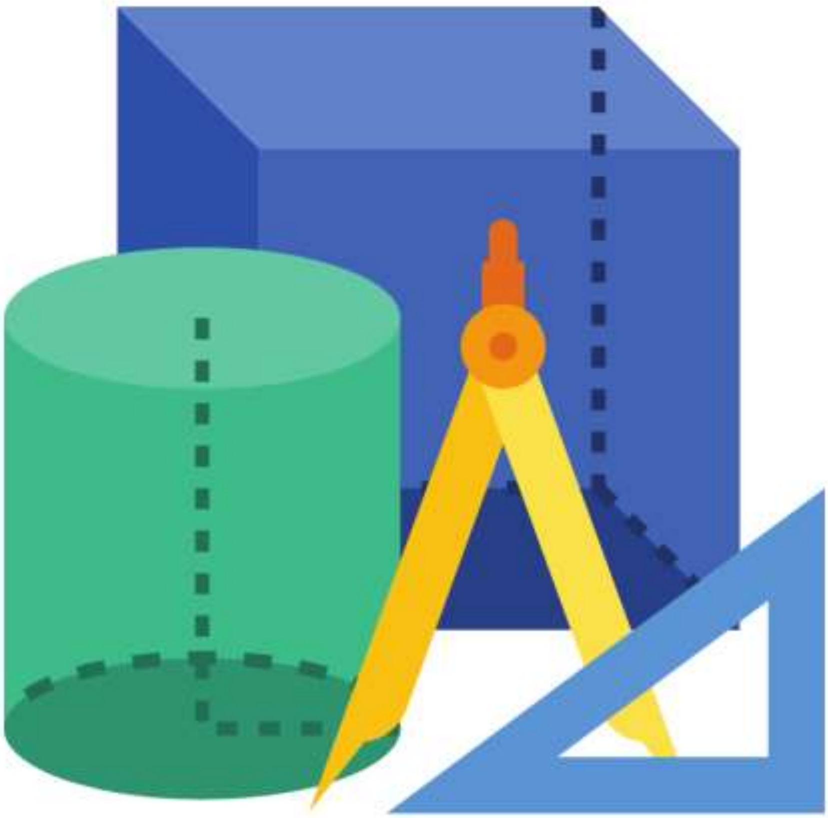
# Basic Geometry & Mathematics



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# BASIC GEOMETRY & MATHEMATICS

**By the end of this chapter you should be familiar with:**

- Rounding
- Percentage Error
- Rules of Exponents
- Rules of Logarithms
- Angles and Triangles
- Circles
- Volume and Surface area of figures

**ROUNDING NUMBERS**

Before understanding how to round numbers we need to know what **approximations** and **estimations** mean. **Approximations** are a value or quantity that is nearly but not exactly correct. **Estimations** are rough calculations of the value, number, quantity, or extent of something. In giving an estimation or approximation, measurements are often **rounded** to some level of accuracy.

Here's the general rule for rounding: If the number you are rounding is followed by 5, 6, 7, 8, or 9, **round** the number up else you **round** it down.

**Example:**

4,827 rounded to the nearest ten is 4,830

4,827 rounded to the nearest hundred is 4,800

4,827 rounded to the nearest thousand is 5,000

## PERCENTAGE ERROR

**Percentage error** expresses as a percentage the difference between an approximate or measured value and an exact or known value.

**Percentage Error:**

**Example:** The report said the carpark held 240 cars, but we counted only 200 parking spaces.

Percentage Error = (|Approximate Value - Exact Value|/|Exact Value|) × 100%

(|240 - 200|/|200|) × 100% = 20%

**Significant figures** are used to express it to the required degree of accuracy, starting from the first non-zero digit.

| Law | Example |
|---|---|
| All non-zero digits are significant | 74818226 has 8s.f<br>123.45 has 5 s.f. |
| All zeros between non-zero digits are significant | 103.05 has 5 s.f<br>780002 has 6 s.f. |
| Zeros to the left of an implied decimal point are non- significant, whereas zeros to the right of an explicit decimal are significant | 23000 has 2 s.f., while 23000.0 has 6 s.f. |
| To the right of a decimal point, all leading zeros are non-significant, whereas all zeros that follow non-zero digits are significant | 0.0043 has 2 sf., while 0.0043000 has 5 s.f. |

# RULES OF EXPONENTS

**Rule 1: When the numbers having the same base are multiplied, add the exponents.**

**Rule 2: When the numbers having the same base are divided, subtract the exponents.**

**Rule 3: Multiply the powers when the numbers are raised by another**

| Law | Example |
|---|---|
| $a^m a^n = a^{m+n}$ | $2^3 2^4 = 2^{3+4} = 2^7 = 128$ |
| $(a^m)^n = a^{mn}$ | $(2^3)^4 = 2^{3.4} = 2^{12} = 4096$ |
| $(ab)^n = a^n b^n$ | $(20)^3 = (2.10)^3 = 2^3.10^3 = 8.1000 = 8000$ |
| $(\frac{a}{b})^n = \frac{a^n}{b^n}$ | $(\frac{2}{5})^3 = \frac{2^3}{5^3} = \frac{8}{125}$ |
| $\frac{a^m}{a^n} = a^{m-n}$ | $\frac{2^5}{2^3} = 2^{5-3} = 2^2 = 4$ |
| $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$ | $\frac{2^3}{2^5} = \frac{1}{2^{5-3}} = \frac{1}{2^2} = \frac{1}{4}$ |

# RULES OF LOGARITHMS

**1)Product Rule:** The logarithm of a product is the sum of the logarithms of the factors. $\log_a xy = \log_a x + \log_a y$

**2) Quotient Rule:** The logarithm of a quotient is the logarithm of the numerator minus the logarithm of the denominator $\log_a(x/y) = \log_a x - \log_a y$

**3) Power Rule:** $\log_a x^n = n\log_a x$

**4) Change of Base Rule:**

$\log_a b = \log_c b / \log_c a$

$\log_a b = 1/\log_b a$

where x and y are positive, and a > 0, a ≠ 1

**Example:** Simplify the following, expressing each as a single logarithm:

a) $\log_2 4 + \log_2 5$

b) $\log_a 28 - \log_a 4$

c) $2 \log_a 5 - 3\log_a 2$

**Solution:**

a) $\log_2 4 + \log_2 5 = \log_2 (4 \times 5) = \log_2 20$

b) $\log_a 28 - \log_a 4 = \log_a (28 \div 4) = \log_a 7$

c) $2 \log_a 5 - 3 \log_a 2 = \log_a 5^2 - \log_a 2^3 = \log_a (25/8)$

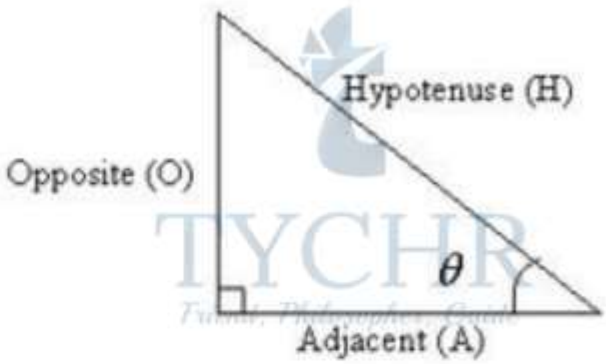
# ANGLES AND TRIANGLES

There are three types of triangles –

**Equilateral:** “equal”-lateral, so they have all equal sides.

**Isosceles:** means “equal legs”, they have two equal “sides” joined by an “odd” side.

**Scalene:** means “uneven” or “odd”, so no equal sides. The sum of the angle of the triangle = 180°




**“Opposite”** is opposite to the angle θ

**“Adjacent”** is adjacent (next to) to the angle θ

**“Hypotenuse”** is the long one

Sin$\theta$ = $Opposite/Hypotenuse$

Cos$\theta$ = $Adjacent/Hypotenuse$

Tan$\theta$ = $Opposite/Adjacent$

**Example:** How tall is the tree (supposing the triangle is the tree)?

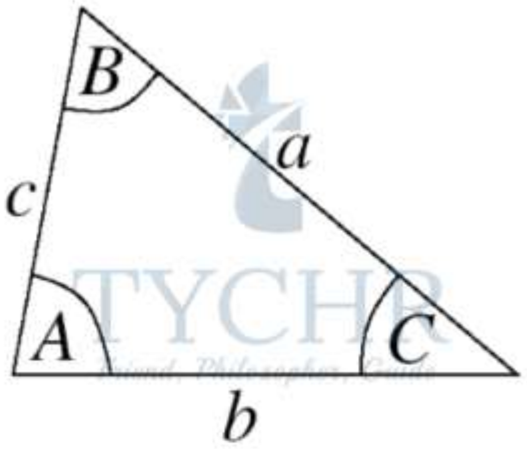
Sin(45°) = $Opposite/Hypotenuse$

0.7071 = Opposite/20

45° Opposite = 14.14 (to 2 decimals)

Rules when the triangle is not right angled:

If a, b and c are the lengths of the sides opposite the angles A, B and C in a triangle,

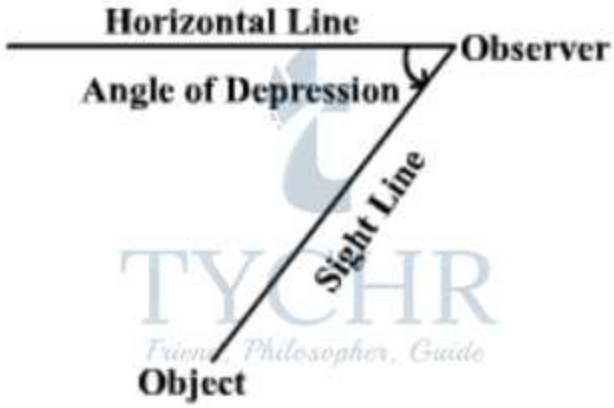



$SinA/a = SinB/b = SinC/c$ **(sine rule)**
**$c^2 = a^2 + b^2 - 2abcosC$ (cosine rule)**
which can also be written as:
**$a^2 = b^2 + c^2 - 2bccosA$**

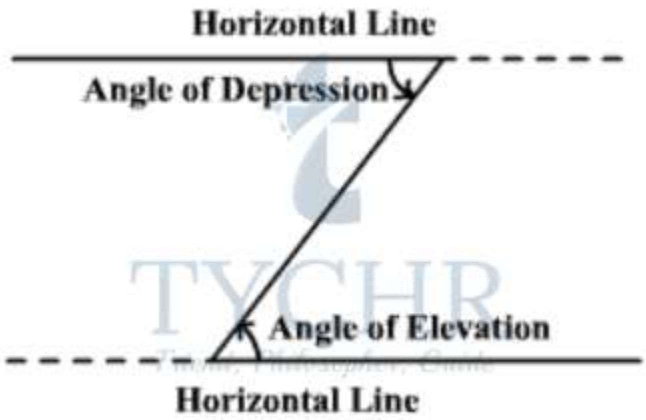
## ANGLE OF ELEVATION AND DEPRESSION:

The term **angle of elevation** denotes the angle from the horizontal upward to an object. An observer's line of sight would be above the horizontal.


The term **angle of depression** denotes the angle from the horizontal downward to an object. An observer's line of sight would be below the horizontal.

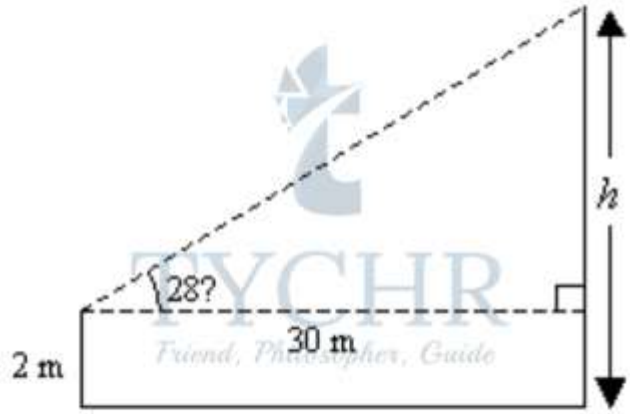



Note that the angle of elevation and the angle of depression are congruent.



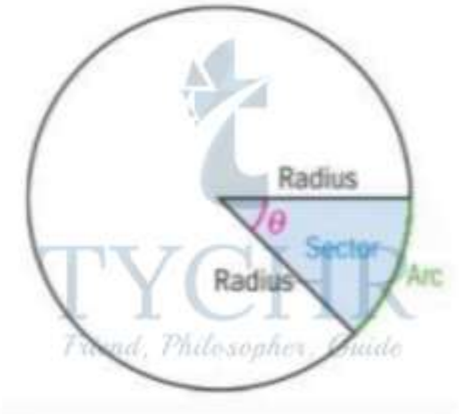

**Example:** A man who is 2 m tall stands on horizontal ground 30 m from a tree. The angle of elevation of the top of the tree from his eyes is 28˚. Estimate the height of the tree.

**Solution:** Let the height of the tree be h. Sketch a diagram to represent the situation.




tan 28˚ = (h-2)/30
h – 2 = 30 tan 28˚
h = (30 ´ 0.5317) + 2 ← tan 28˚ = 0.5317 = 17.951
The height of the tree is approximately 17.95 m.

## CIRCLES

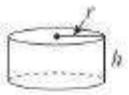
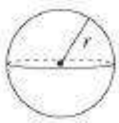
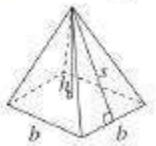
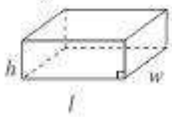



Area of a circle = $\pi r^2$
Surface area of a circle = $2\pi r$

The **length of arc** of a circle with radius "r" and central angle "θ" (in degrees) is = = $\theta/360 \times 2\pi r$
The **area of the sector** with central angle "θ" is = = $\theta/360 \times \pi r^2$

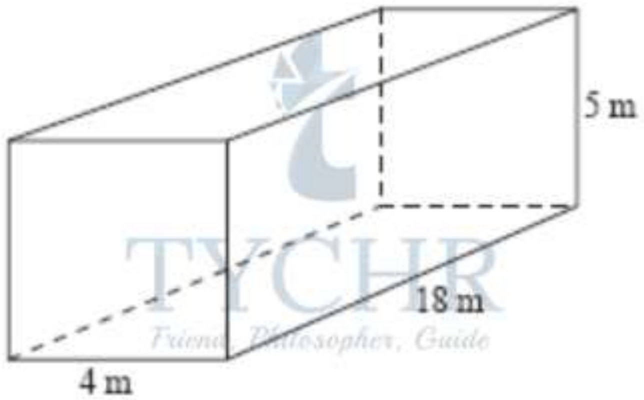
# THREE-DIMENSIONAL GEOMETRY

## VOLUME OF 3-D SOLIDS

| Geometric Solid | Surface Area | Volume |
|---|---|---|
| Cylinder  | $A_{top} = \pi r^2$<br>$A_{base} = \pi r^2$<br>$A_{side} = 2\pi rh$<br>$SA = 2\pi r^2 + 2\pi rh$ | $Volume = \pi r^2 h$ |
| Sphere  | $SA = 4\pi r^2$<br>**or**<br>$SA = \pi d^2$ | $Volume = \frac{4\pi r^3}{3}$ |
| Cone  | $A_{side} = \pi rs$<br>$A_{base} = \pi r^2$<br>$SA = \pi r^2 + \pi rs$ | $Volume = \frac{\pi r^2 h}{3}$ |
| Square-Based Pyramid  | $A_{triangle} = \frac{1}{2}bs$ (for each triangle)<br>$A_{base} = b^2$<br>$SA = 2bs + b^2$ | $Volume = \frac{b^2 h}{3}$ |
| Rectangular Prism  | $SA = wh + wh + lw + lw + lh + lh$<br>**or**<br>$SA = 2(wh + lw + lh)$ | $Volume = lwh$ |
| General Right Prism | $SA =$ the sum of the areas of all the faces | $Vol =$ area of base of prism $\times$ height of prism |
| General Right Pyramid | $SA =$ the sum of the areas of all the faces | $Vol = \frac{\text{area of base of pyramid} \times \text{height of pyramid}}{3}$ |

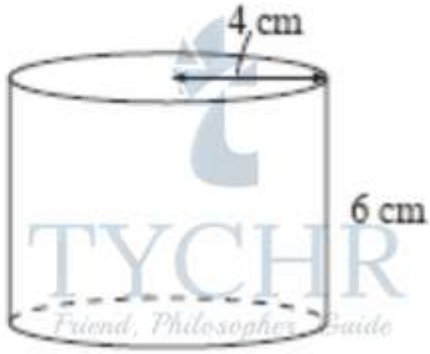
Where r=radius, h=height, w=width, l=length, b=base length

**Example 1:**




- Calculate the *volume*of the cuboid shown. Volume = 4 × 18 × 5 = 360 $m^3$
  Surface area = (2 × 4 × 18) + (2 × 4 × 5) + (2 × 5 × 18)
  = 144 + 40 + 180
  = 364 $m^2$
- Calculate the *surface area*of the cuboid shown.

**Example 2:**




Calculate the *volume* and total *surface area* of the cylinder shown.

Volume = $\pi r^2 h = \pi \times 4^2 \times 6 = 96\pi$

*=301.5928947 $cm^3$*

*=301.5928947 $cm^3$*

*=302 $cm^3$ (to 3 significant figures)*

Area of curved surface = $2\pi rh = 2 \times \pi \times 4 \times 6$

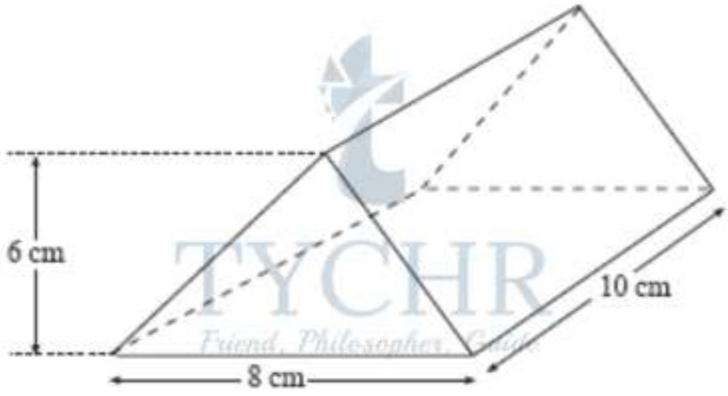
= $48\pi$

*= 150.7964474 $cm^2$*

Area of each end = $\pi r^2 = \pi \times 4^2$
= $16\pi$
= 50.26548246 $cm^2$

Total surface area = 150.7964474 + (2 × 50.26548246)
= 251.3274123 $cm^2$
= 251 $cm^2$ (to 3 significant figures)

**Example 3:**




Calculate the *volume* of this prism.
Area of end of prism = 12 × 8 × 6
= 24 $cm^2$

Volume of prism = 24 × 10
= 240 $cm^3$

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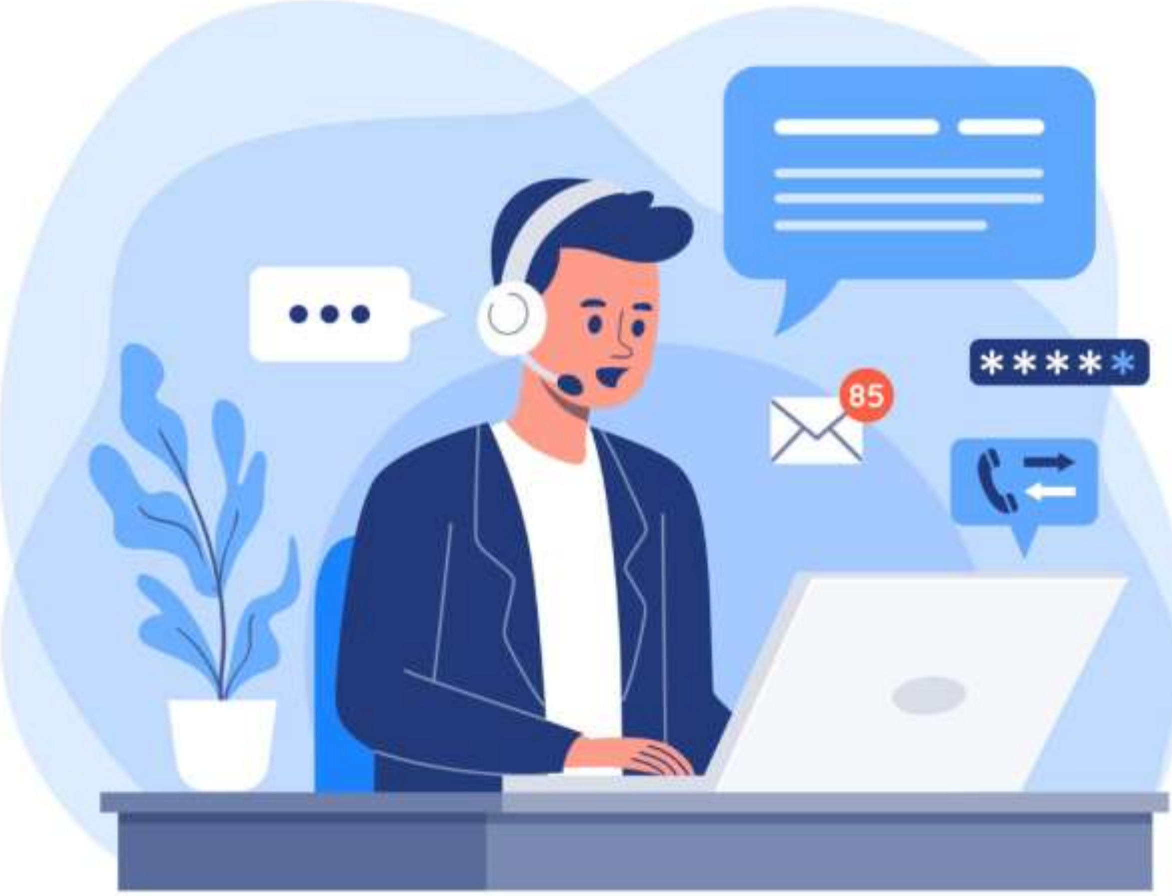


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